Internet Appendix for "Debt Correlations in the Wake of the Financial Crisis: What are Appropriate Default Correlations for Structured Products?"

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This set of appendices details the data used (Appendix A) and outlines the estimation procedure for the unobservable frailty path (Appendix B).

Appendix A

Our large sample of detailed CDO information is collected from the first available surveillance reports obtained from both S&P and Moody's, and supplemented by pre-sale and new issue reports. We require the number of obligors from S&P's surveillance report when computing default correlations. Our sample consists of CDOs backed by bond and loan collateral. Further details of a data set collected in a similar fashion can be found in Griffin, Nickerson, and Tang (2013).

Appendix B

This appendix briefly outlines the steps used to estimate the unobservable frailty path. We follow the procedure laid out in Duffie, Eckner, Horel and Saita (2009), which should be consulted for a more thorough discussion of the estimation procedure. The E-M algorithm involves iterating between two steps until convergence of the parameters is achieved. Let Z denote the set of firm defaults observed. To begin, we initialize the parameter set $\Theta = \{\alpha, \beta, \eta, \kappa\}$ to $\Theta^{(0)} = \{\hat{\alpha}, \hat{\beta}, 0.10, 0.05\}$ where $\hat{\alpha}$ and $\hat{\beta}$ are the MLE estimates of the model described in equation (1) when omitting the frailty component. We then iterate between an expectation step (E) and a maximization step (M), starting with the first iteration ($i = 1$) in the following manner:

E Step

Draw N sample paths for the un-observed frailty path Y from the conditional probability of Y given the parameter set $\Theta^{(i-1)}$ using the following Gibbs sampler:

1. Initialize the frailty path, such that $\{Y_1^{(0)}, \ldots, Y_T^{(0)}\} = 0$
2. Set $k = 0$
3. Draw a candidate path $y = \{y_1, \ldots, y_T\}$ such that $y_t \sim N\left(Y_t^{(k)}, 4\right)$
4. Compute the acceptance probability for each period of the candidate path, $\alpha_1, \ldots, \alpha_T$ in the following manner:

$$\alpha_t = \min\left(\frac{L\left(\Theta|Z, Y_{(-t)} = Y_{-t}^{(k)}, Y_t = y_t\right)}{L\left(\Theta|Z, Y_{(-t)} = Y_{-t}^{(k)}, Y_t = Y_t^{(k)}\right)}, 1\right) \tag{1}$$

5. Generate the next draw of the frailty path using the acceptance probabilities calculated in step 5 as follows:

$$Y_t^{(k+1)} = \begin{cases} y_t, & \text{if } u_t < \alpha_t \\ Y_t^{(k)}, & \text{otherwise} \end{cases} \quad u_t \sim U(0,1) \tag{2}$$

6. Replace $k = k + 1$. Return to step 3.

   Note: we use a burn-in sample of 1,000 draws which we discard before drawing paths from the Gibbs sampler.

<u>M Step</u>

Compute the parameter set $\Theta^{(i)}$ which maximizes the expected log-likelihood of equation (3) using the sample frailty paths, $\{Y^{(1)}, \ldots, Y^{(N)}\}$:

$$\begin{aligned} \Theta^{(i)} &= \arg\max_{\Theta} E\left[\log L\left(\Theta|Z, Y\right)\right] \\ &= \arg\max_{\Theta} \frac{1}{N}\sum_{i=1}^{N} \log L\left(\Theta|Z, Y^{(i)}\right) \end{aligned} \tag{3}$$

We then check for convergence of the parameter set. If not achieved, replace $i = i + 1$ and repeat both the E and M steps.

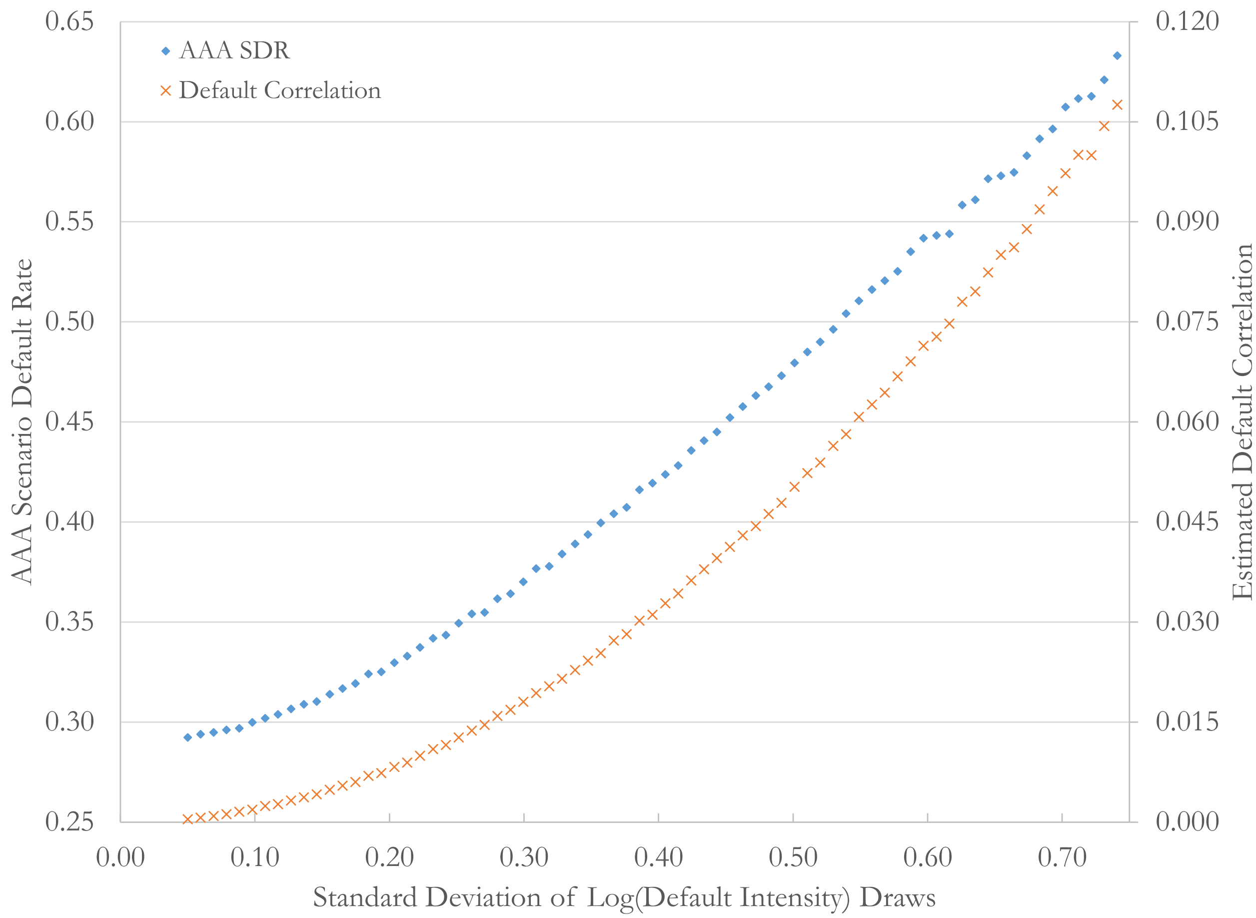

Figure IA.1
Tail Risk and Default Correlation

This figure reports the scatter-plot of AAA Scenario Default Rates (blue diamonds) and estimated default correlations (orange crosses) when drawing a common default intensity for all assets from log-normal distributions with varying standard deviations. In all simulations, default intensities are perfectly correlated across all assets. The probability of default for each asset is fixed at 0.20, roughly equal to the default probability observed in our sample of CDOs reported in Table 1. The number of equal sized assets in the hypothetical CDO's collateral pool is set to 100.

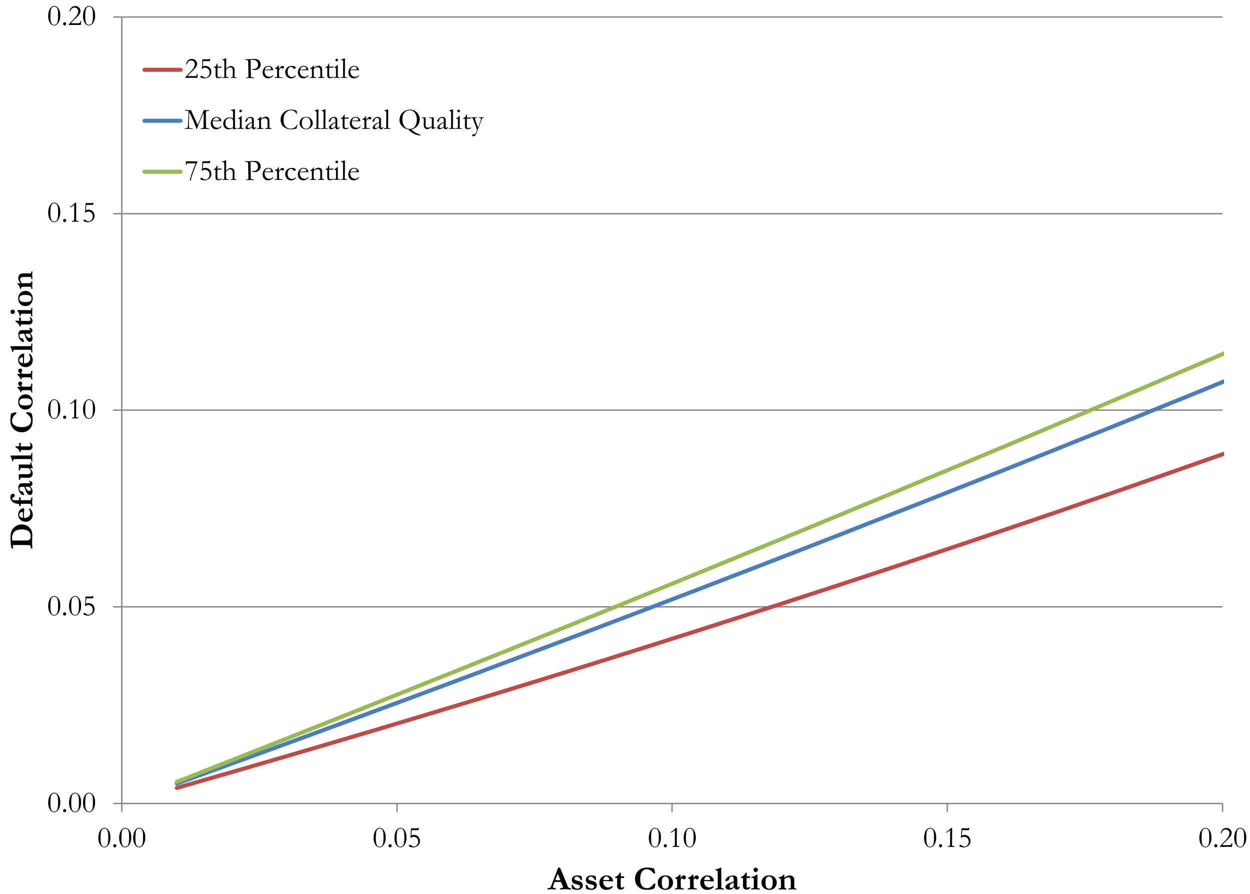

Figure IA.2
Asset Correlation versus Default Correlation
This figure illustrates the relationships between asset correlation and default correlation. The x-axis reports the asset correlation used as an input in the Gaussian Copula and the y-axis reports the average pair-wise default correlation from the simulated asset defaults. A pool of 131 assets, which corresponds to the median number of obligors in our sample of CDOs, was simulated 100,000 times for each asset correlation assumed. Reported are the mappings from asset correlation to default correlation when each assets probability of default is set to the 25th, 50th, and 75th percentiles of the average underlying collateral probability of default in our sample.

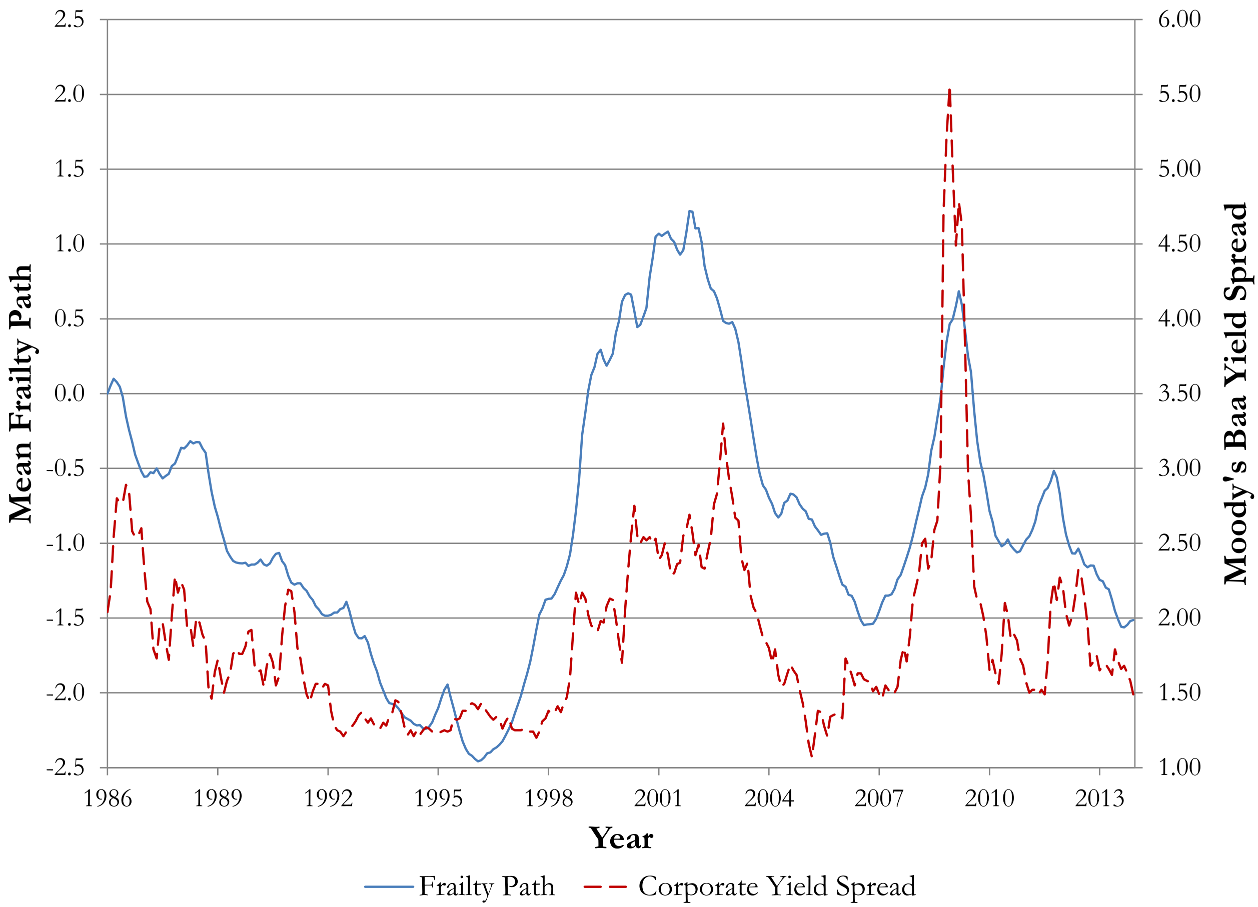

Figure IA.3
Corporate Bond Yield and Estimated Frailty Path
This figure illustrates the conditional mean of the frailty path from a hazard model fitted using the firm's credit rating lagged by 1 month (solid blue line) and the monthly corporate credit yield spread (dashed red line). 4,800 paths were drawn from a Gibbs sampler using the estimated coefficients from the fitted frailty and rating transition models reported in Table 2. The path has been scaled by the appropriate scaling parameter, η. *Corporate Yield Spread* is the Moody's Seasoned Baa Corporate Bond Yield minus the 30-year Treasury Constant Maturity Rate, both provided by FRED.